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2001 J. Phys.: Condens. Matter 13 L135

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J. Phys.: Condens. Matter **13** (2001) L135–L140 www.iop.org/Journals/cm PII: S0953-8984(01)19554-8

LETTER TO THE EDITOR

Crack formation in two-dimensional annular networks

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Received 29 November 2000, in final form 2 January 2001

Abstract

We used the random-fuse model to compute the topological features of localized cracks in an annular two-dimensional geometry. Depending on the type of disorder in the resistivity of the fuses, the results follow the observations encountered in the literature for square lattices with periodic boundary conditions or have a different behaviour explained by a small disorder in the resistivities. Finally, finite-size effects play an important role in this type of geometry.

Fracture has been widely studied since 1980 [1, 2]. One model for analysing numerically cracks on brittle fracture is the random-fuse model (RFM) [3]. Fuses are placed on a lattice and an electrical potential difference is applied to them (or equally, a given current is injected at some location). Depending on the resistance threshold of the fuses, some burn out and become insulators while some do not.

The RFM is, in two dimensions, an exact recasting of the mechanical problem of constraint application on a material as an electrical problem with a given electrical potential difference [1]. It has also been used in three dimensions [4] and the results compared to two-dimensional ones. This model allows one to compute the behaviour of fracture from a topological point of view. It corresponds to a quasi-static phenomenon and may be investigated in a deterministic manner.

We will consider here the special case of the RFM where some disorder has been applied to the fuse resistances. Depending on this random distribution, the burned-out fuses are either localized on a crack or spread all over the numerical material.

All RFMs in the literature used cubic (square in two dimensions) lattices with periodic boundary conditions [1, 3, 4].

We will here impose an annular geometry on our boundaries on a triangular lattice; i.e. the triangular lattice will be bounded by two circles of different radii.

Depending on the disorder in the resistivity of the fuses, this type of geometry leads to results known generally from the literature for 2D cubic lattices with periodic boundary conditions or to different results which are less frequently encountered in the literature. The differences from cubic lattices here are the finite-size scale of the geometry and the geometry itself.

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The model is a RFM on a triangular lattice with the nearest-neighbouring sites connected by bonds, taken to be electrical fuses with identical resistance but whose burn-out thresholds are disordered.

The boundaries of our triangular lattice are an outer circle of radius R_{out} and an inner circle of radius R_{in}. A fuse conducts until the current that it carries, i, exceeds the burn-out (breaking) threshold i_c, at which point it becomes irreversibly an insulator. We apply a constant potential difference between the circle R_{in} (potential $V = 1$) and the circle R_{out} (potential $V = 0$). The equations to be solved are the usual current equations (Kirchhoff's equations) on the lattice, which can be easily solved using the conjugate-gradient method [5].

The breaking thresholds were assigned by generating random numbers uniformly distributed on the unit interval and raising them to a power D. This corresponds to a cumulative probability distribution $P(i_c) = i_c^{1/|D|}$ when $D > 0$ and $P(i_c) = 1 - i_c^{-1/|D|}$ when $D < 0$. The smaller the value of $|D|$, the smaller the disorder. The two-dimensional fuse model was shown to exhibit distinct classes of fracture behaviour according to the value of D [6]. For small values of $|D|$, a macroscopic crack starts developing early in the fracture process, while when $|D|$ is large, a cloud of disconnected micro-cracks (blown fuses) develops before they coalesce into a final macroscopic crack.

We took a constant value for R_{out}, namely $R_{out} = 40$, and seven different values for R_{in}, namely $R_{in} = 5, 10, 15, 20, 25, 30, 35$. Our stopping condition is for the residual current to be less than 10^{-13}. We also used two values for D: 0.3 and -0.3, for which the fracture is known to be localized. To analyse the effect of the value of D on the results, we also used $D = 0.5$ and $D = -0.5$ to compare with the previous values for the same R_{out}, but only for $R_{in} = 5, 10, 20$. Finally, for each set of values R_{in}, R_{out} and D, we averaged the results over 20 different realizations.

The fractures were isolated on our crack images by using an image analyser program. Once the cracks were isolated, we computed the number of pixels of the cracks which is proportional to the number of micro-cracks (which do not overlap). From the coordinates of each micro-crack composing the fracture, we calculated the characteristic topological features of the crack. For an example of fracture, see figure 1.

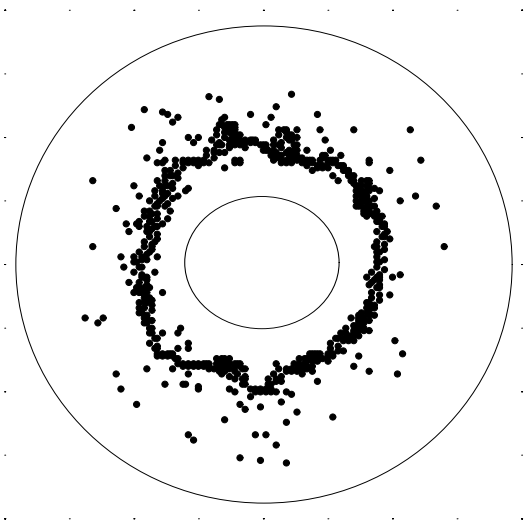


Figure 1. An example of fracture for $R_{out} = 40$, $R_{in} = 20$ and $D = -0.3$. The scale has been multiplied by two for better visibility

It has been observed in the literature on the RFM that the crack lengths in square lattices scale with the typical dimension of the lattice R. For example, in three dimensions, the crack length ℓ follows a power law $\ell = R^{\alpha}$ where $\alpha = 2.13$ for low disorder and $\alpha = 2.7$ for high disorder [7].

In our case and for $D = 0.3$ or $D = -0.3$, we found that

$$\ell = R^{\beta} \tag{1}$$

with $\beta = 0.85 \pm 0.005$. We can say that the length of the fracture is approximatively proportional to the outer radius R_{out} (see figure 2(a) and figure 2(b)), while for the values of $D = 0.5$ and $D = -0.5$, the scaling law is

$$\ell = R^{\delta} \tag{2}$$

with $\delta = 1.12 \pm 0.015$. We can say in this case that, as for three-dimensional systems, systems with $D = 0.3, -0.3$ are low-disorder systems and systems with $D = 0.5, -0.5$ are high-disorder systems.

Another characteristic is the Hurst exponent H or roughness exponent of the fracture. It is defined by

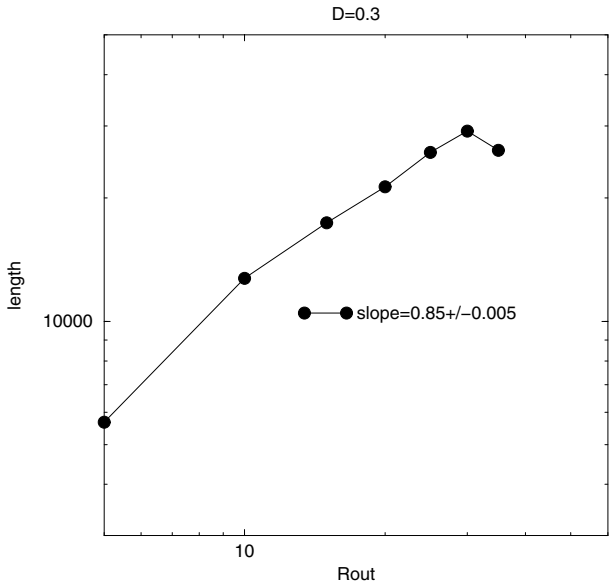
$$C(r) = 2(\langle [h(\boldsymbol{r}_0)]^2 \rangle - \langle h(\boldsymbol{r}_0) \rangle^2) = 2\sigma^2 \propto \ell^{2H} \tag{3}$$

where $C(r)$ is the autocorrelation function of the height of the crack (i.e. the distance from the centre of the disk) $h(\boldsymbol{r}_0)$ with respect to the average fracture location and σ is the crack width. Another way to compute the Hurst exponent is to find the difference between the maximum distance of the crack from the centre of the circles and the minimum one. This leads to a power law in this difference versus the size of the inner circle whose exponent is also the Hurst exponent. And, finally, the third way to compute the Hurst exponent is to take the average of the two eigenvalues of the moment-of-inertia tensor of the fracture. Here again, this leads to a power behaviour whose exponent is the Hurst exponent.

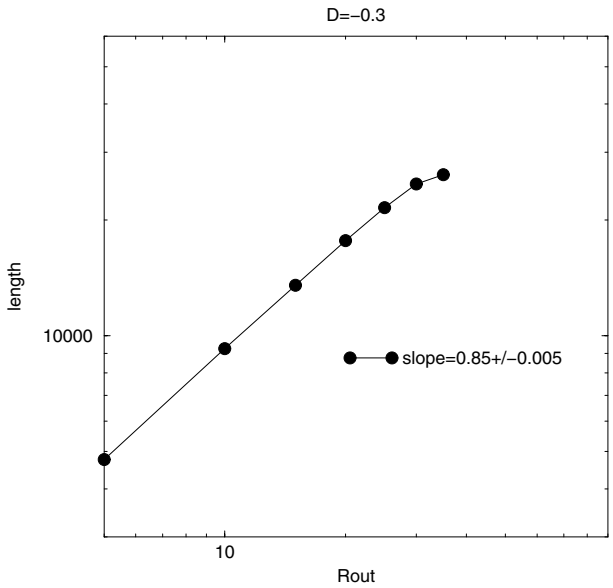
The results for H in the case of $D = -0.3$ are $H = 0.7 \pm 0.05$ for the three different computation methods. For $D = 0.3$, $H = 0.6 \pm 0.07$, also for the three computation methods. We show these results in figure 3(a) and figure 3(b), the Hurst exponent being deduced from the slope in a log–log plot.

In the cases of $D = 0.5$ and $D = -0.5$, what is interesting, and what seems inconsistent with the uniqueness of the value of δ, is the two values of H that we find here. For a disorder D equal to -0.5, we have an average value of $H = 0.43 \pm 0.02$, while for a disorder equal to 0.5, $H = 0.74 \pm 0.02$. We stress that we used the same metrics to compute H as for $D = 0.3, -0.3$. The apparent contradiction between H and the length (ℓ) scaling factor may be explained if one looks at the disorder. For $D = -0.5$ the crack expands without any contribution from micro-cracks. The resulting fracture is then smoother than for the case of disorder $D = 0.5$. In this latter case, micro-cracks are added to the fracture and thus increase the roughness of the fracture. So the length ℓ of the crack can be the same on average for the two types of disorder, while at very small scales the roughness for $D = 0.5$ is double that for $D = -0.5$.

The type of disorder in the resistivity of the fuses plays an important role, which is linked to the finite-size effects. Indeed, for $D = 0.3, -0.3$, the results follow the same rules as for two-dimensional computation for the RFM on square lattices with periodic boundary conditions; i.e. the Hurst exponent is only slightly different for the two cases of disorder. In contrast, we may say that for the case of disorder $D = 0.5$, the fracture is entirely disorder driven—i.e., that the roughness of the fracture arises essentially from the randomly distributed micro-cracks which appear in the neighbourhood of the macroscopic crack—while for disorder $D = -0.5$, the fracture shows a roughness which may be compared to that of small-scale fractures. The value of H for $D = 0.5$ may be compared to the theoretical value of 2/3 for the roughness



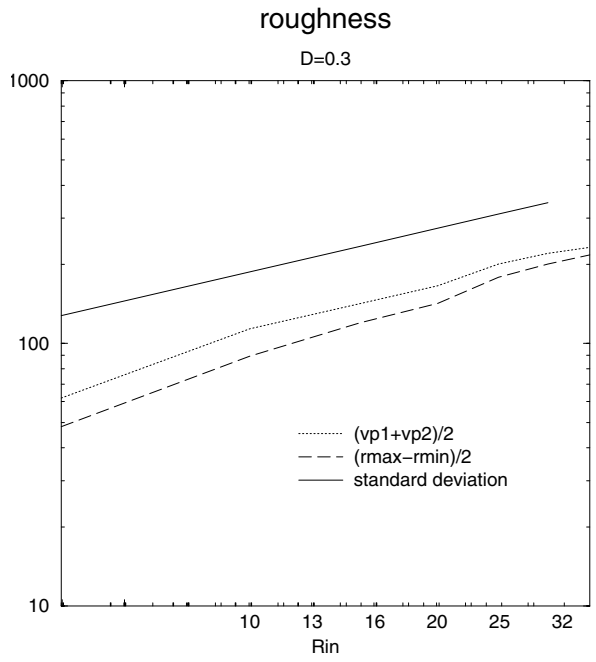
(a)



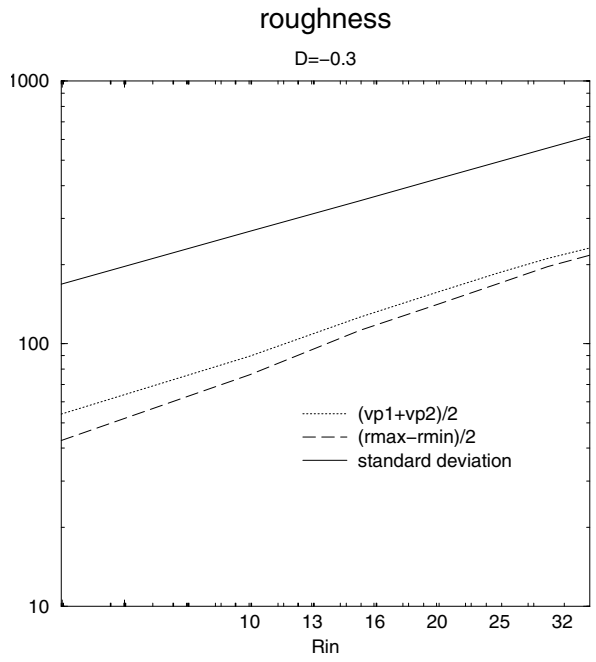
(b)

Figure 2. (a) A log–log plot of the length of the fracture versus the outer radius for $D = 0.3$. The slope 0.85 has been computed by means of a power regression, excluding the last point for which the finite-size effects are too large. (b) A log–log plot of the length of the fracture versus the outer radius for $D = -0.3$. Here again, in the power regression used to compute the slope of 0.85, the last point has been excluded.

exponent in two dimensions (as for $D = 0.3, -0.3$), while for the value $H = 0.43 \pm 0.02$ for $D = -0.5$ we have to recall a result obtained by Schmittbuhl and Vilotte [8]: they constructed a model which simulates a front constraint to move in a plane with small disorder. In this



(a)



(b)

Figure 3. (a) The case of disorder $D = 0.3$. The Hurst coefficient (equal to 0.6) has been computed by constructing a power regression on this log–log plot of (i) the standard deviation of the crack, (ii) the average of the two eigenvalues of the moment-of-inertia tensor and (iii) the difference between the maximum distance to the centre of the circles and the minimum distance. (b) The case of disorder $D = -0.3$. The same methods have been used as for (a). Here, the Hurst coefficient is equal to 0.7.

case, the prediction for the Hurst exponent is of order 1/3. Another hypothesis is that for the value $D = 0.5$ we observe pseudo-brittle fracture (i.e. a more gradual fracture process of the material) as described in the article of Sahimi and Goddard [9]. Indeed, they used the same random disorder for the distribution of the current threshold as for the fuses (but produced mechanically). We observed, as they do, many cracks spread all over the system and not belonging to the percolating fracture. Moreover, in a square lattice with periodic boundary conditions, these micro-cracks do not appear as numerous as here, with the same D. This may explain the different value of the Hurst exponent for this type of disorder. We have to say finally that, when $R_{in} = 35$, i.e. is very close to $R_{out} = 40$, the finite-size effects are large and the power laws observed for the other values of R_{in} are no longer valid.

As a conclusion to this letter, we may say that geometry plays an important role in the behaviour of the fracture, as well as the random distribution of the current threshold of the fuses in the RFM. We recover the same results as for square lattices for small disorders, i.e. brittle fracture behaviour. But, due to the confined geometry, on increasing the disorder the behaviour becomes closer to pseudo-brittle fracture, while for square lattices the fracture behaviour remains brittle for all values of the disorder exponent $|D| < 1$. So, what is important is that even for numerical models such as the RFM, which compute brittle fractures, the behaviour of the crack topological features is not universal for confined geometries.

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